Some Solved Problems and Hints for Week Six

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G2.6#1 For parts (c) and (d), use the appropriate DeMorgan law.

G2.6#5 Recall that $\alpha \in \mathbf{R}$ and $\alpha \geq 0$ then $[-\alpha, \alpha] = \{x | -\alpha \leq x \leq \alpha]$, a subset of $\mathbf{R}$. Determine each of the following:

(a) $\bigcup_{\alpha>0}[-\alpha, \alpha] = \mathbf{R}$. The proof is in two parts. Showing that the union is a subset of the reals is trivial, since each of the intervals is a subset of the reals. For the converse, pick an arbitrary real number, say x, and show that x is contained in at least one of the intervals in question.

(b) $\bigcap_{\alpha>0}[-\alpha, \alpha] = \{0\}$. Obviously 0 is contained in the intersection, since 0 is an element of each of the intervals. Given a nonzero real number, say y, you should be able to construct an interval of the form $[-\alpha, \alpha]$ that does not contain y to finish the proof.

(c) $\bigcap_{\alpha\geq 5}[-\alpha, \alpha] = [-5, 5]$.

G2.6#6 For each integer n, let M_n denote the set of all integer multiples of n. Determine each of the following:

(a) $\displaystyle\bigcap_{n\in\mathbf{N}} M_n = M_1 = \mathbf{Z}$

(b) $\displaystyle\bigcup_{n\in\mathbf{N}} M_n = M_0 = \mathbf{0}$

(c) $\displaystyle\bigcup_{p \text{ prime}} M_p = \mathbf{Z} - \{-1, 1\}$, since every integer except 1 and -1 is a multiple of at least one prime.

(f) $\displaystyle\bigcup_{n\in M_5} M_n = M_5$.

G2.6#8 Describe each of the following as an indexed family of sets. Here Π denotes the coordinatized plane.

(a) The family of closed intervals of unit length on the real line. For each $i \in \mathbf{R}$, let $S_i = [i, i+1]$, and let $S = \{S_i\}_{i\in\mathbf{R}}$.

(c) The family of all circles in Π of radius 1 (unit circles). Our index set is $\mathbf{R} \times \mathbf{R}$. For each coordinate pair (a, b), let $C_{a,b}$ denote the unit circle centered at (a, b), i.e.,
$C_{a,b} = \{(x, y)|(x-a)^2 + (y-b)^2 = 1\}$. Let $C = \{C_{a,b}\}_{a,b\in\mathbf{R}}$

(e) The family of all lines in Π with y-intercept 5. These differ from one another in slope only. For each real number m, let $l_m = \{(x, y)|y = mx + 5\}$, and let $L = \{l_m\}_{m\in\mathbf{R}}$.

MA/Ra, February 19, 2004